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Temperature Dependent Viscosity Effect on Buoyancy-Surface Tension Driven Convection in a Rotating Ferrofluid Layer and Submitted to Robin Thermal Boundary Conditions

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Abstract— Combined effect of buoyancy and surface tension forces in a rotating ferrofluid layer heated from below is studied using linear stability analysis of the Navier-Stokes equations supplemented by Maxwell's equations and the appropriate magnetic force subjected to temperature dependent viscosity. The lower boundary is considered to be rigid at constant temperature, while upper boundary free open to the atmosphere is flat and subject to a Robin-type of thermal boundary condition. The weighted residual Galerkin technique is employed to extract the critical stability parameters numerically. It is shown that convection sets in oscillatory motions provided that the Prandtl number ($\Pr$) is less than unity. A mechanism for suppressing or augmenting Bénard–Marangoni ferroconvection by Coriolis force (Ta), temperature dependent viscosity (η), Biot number (Bi), magnetic Rayleigh number (R_m) and nonlinearity of fluid magnetization (M_3) is discussed in detail. It is found that the onset of Bénard–Marangoni ferroconvection is delayed with an increase in η, Ta, Bi but opposite is the case with an increase in M_3, R_m. Further, increase in M_3 and decrease in η, Ta and Bi is to decrease the size of the convection cells. A few results are known as recovered to special cases

Index Terms— ferrofluid; heat transfer coefficient; rotation; surface-tension; temperature dependent viscosity; heat transfer coefficient; Galerkin technique; Convection cell.

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1 Introduction

Ferrofluids (magnetic fluids) are commercially manufactured colloidal liquids usually formed by suspending mono–domain nano particles (their diameter is typically 10 nm) of magnetite in non–conducting liquids like heptanes, kerosene, water, etc. and they are also called magnetic nano fluids. The ferrofluid is a type of functional fluid whose flow and energy transport processes may be controlled by adjusting an external magnetic field, which makes it find a variety of applications in various fields such as electronic packing, mechanical engineering, aerospace, bioengineering and thermal engineering. An authoritative introduction to this fascinating subject along with their applications is provided in Odenbach [1], Rosenwieg [2] and Shliomis [3].

The magnetization of ferrofluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is also known as Bénard-ferroconvection Finlayson [4], Penfield [5], Rosenweig [6], Sekhar [7]. Convective instability in a ferrofluid layer can also be induced by surface tension forces provided it is a function of temperature and/or concentration. In view of the fact that heat transfer is greatly enhanced due to convection, the ferroconvection problems offer new possibilities for new applications in cooling of motors, loud speakers, transmission lines, and other equipment where magnetic field is already present. If the ferrofluid layer has an upper surface open to atmosphere then the instability is due to the combined effects of buoyancy as well as temperature–dependent surface tension forces, known as Bénard – Marangoni ferroconvection. A limited number of literatures have addressed this type of instability problem in a horizontal ferrofluid layer. Linear and nonlinear stability of combined buoyancy and surface-tension effects in a ferrofluid layer heated from below has been analyzed by Qin and Kaloni [8]. The Bénard–Marangoni convection problems of ferrofluid layer heated from below under various assumptions is studied by many authors Odenbach [9], Hennenberg et al. [10], [11], [12], Idris and Hashim [13], Nanjundappa et al. [14], [15], [16], [17], Shivakumara et al. [18]. The effect of viscosity variations on the onset of Bénard –Marangoni ferroconvection in a horizontal layer of ferrofluid was investigated by Nanjundappa et al. [19], [20]. Recently, Sekhar et al. [21] have studied the effect of variable viscosity on thermal convection in Newtonian ferromagnetic liquid by considering different forms of boundary conditions.

The study of fluids in rotation is itself an interesting topic for research. Ferrofluids are known to exhibit peculiar characteristics when they are set to rotation. Gupta and Gupta [22] have studied the convective instability in a rotating layer of ferrofluids between two free boundaries. The effect of rotation on thermo–convective instability of a horizontal layer of ferrofluid confined between stress–free, rigid–paramagnetic and rigid–ferromagnetic boundaries was discussed by Venka-

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tasubramanian and Kaloni [23]. Thermal convection in a rotating layer of a magnetic fluid is discussed by Auernhammerand Brand [24]. Vaidyanathan and Sekar [25] have studied linear stability analysis for the effect of MFD viscosity on the onset of ferroconvection in rotating medium. The weakly nonlinear instability of a rotating ferromagnetic fluid layer heated from below is studied by Kaloni and Lou [26].

Shivakumara and Nanjundappa [27] have studied the effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Prakash and Gupta [28] have proved analytically that the complex growth rate of an arbitrary oscillatory motion of growing amplitude in ferroconvection rotating ferrofluid layer with MFD viscosity. Nanjundappa et al. [29] have studied the combined effect of rotation and magnetic field dependent viscosity on Bénard-Marangoni convection in a ferrofluid layer.

In view of the fact that rotation gives rise to interesting practical situations, the object of this paper is to study the combined effect of rotation and surface tension force on the linear stability of Bénard–Marangoni ferroconvection. In this study, the lower rigid boundary is considered to be isothermal and the upper non–deformable free boundary is insulating to temperature perturbations. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique. A comparative study is conducted to analyze on the onset of convection and also with the other works under the limiting conditions

2 Mathematical Formulation

We consider an incompressible ferromagnetic fluid bounded by two nonmagnetic horizontal plates at $0 \le z \le d$ in presence of vertical magnetic field $\vec{H} = H_0\hat{k}$ and is perpendicular to the boundary. The lower and the upper boundaries are maintained at constant but different temperatures T_0 and $T_1 < (T_0)$ respectively. The layer is rotating uniformly about its vertical axis with angular velocity $\vec{\Omega}=\Omega\hat{k}$, which is bounded below by a rigid–isothermal surface and above by a non–deformable free–insulating surface with thermally constrained by a Robin boundary condition.

The stream of Benard-Marangoni convection for thermocapillary forces, buoyancy forces and viscous forces is due to the linearly temperature dependent surface tension (σ) and viscosity (η), respectively. The following relations are considered:

$$\sigma = \sigma_0[1-\sigma_T(T-T_0)] \tag{1}$$

$$\mu = \mu_0[1-\eta(T-T_0)] \tag{2}$$

where $\sigma_T = (\partial\sigma/\partial T)_{T=T_0}$ the rate of change of surface tension with temperature and σ_0 the unperturbed value surface tension with temperature, μ_0 and η are positive constants. For an incompressible, ferromagnetic fluid in the presence of uniform magnetic field, the basic equations solved our simulation the Navier-Stokes equations for fluid flow, including the viscous force $\vec{F}_v = \nabla\cdot[\eta\,(\nabla\vec{V}+(\nabla\vec{V})^T)]$, magnetic force $\vec{F}_m = \mu_0(\vec{M}\cdot\nabla)\vec{H}$ and Coriolis force acceleration $\vec{F}_C = 2\rho_0(\vec{V}\times\vec{\Omega})$. The complete sets of equations are

$$\nabla\cdot\vec{V} = 0 \tag{3}$$

$$\rho_0\left[\frac{\partial\vec{V}}{\partial t}+(\vec{V}\cdot\nabla)\vec{V}\right] = -\nabla p + \rho_0[1-\alpha_t(T-\bar{T})]\vec{g} + \vec{F}_v+\vec{F}_m+\vec{F}_C+\rho_0\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right)/2 \tag{4}$$

$$\left[\rho_0 C_{V,H} - \mu_0\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt} = k_t\nabla^2 T \tag{5}$$

$$\nabla\cdot(\vec{M}+\vec{H}) = 0,\ \nabla\times\vec{H} = 0 \text{ or } \vec{H} = \nabla\varphi \tag{6}$$

$$\vec{M} = [M_0 + \chi(H-H_0) - K(T-\bar{T})](\vec{H}/H) \tag{7}$$

where, $\vec{V} = (u, v, w)$ is the fluid velocity, t the time, $\vec{g}$ the acceleration of gravity, the coefficient $\mu_0 = 4\pi\times10^{-7}\,\text{Henry}\,m^{-1}$ the magnetic constant, $C_{V,H}$ the specific heat capacity at constant volume and magnetic field, k_t the thermal conductivity, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ the Laplacian operator, $H = |\vec{H}|$ and $M = |\vec{M}|$. The centrifugal force can be combined with static pressure p and its defined by modified fluid pressure term:

$$\left[(1+\eta z)(D^2-a^2)+2\eta D-\omega\right](D^2-a^2)W = Ta^{1/2}D\xi + R_t a^2\Theta - R_m a^2(D\Phi-\Theta) \tag{8}$$

$$(D^2 - a^2 - \Pr\omega)\,\Theta = -(1-M_2)W \tag{9}$$

$$(D^2 - a^2 M_3)\,\Phi - D\Theta = 0 \tag{10}$$

$$\left[(1+\eta z)(D^2-a^2)+\eta D-\omega\right]\xi = -\sqrt{Ta}\,DW \tag{11}$$

Here, gravity thermal Rayleigh number $(R_t = \alpha_t g\beta d^4/\kappa\mu)$, magnetic number ($M_1 = \mu_0 K^2\beta/(1+\chi)\alpha_t\rho_0 g$), magnetic thermal Rayleigh number ($R_m = \mu_0 K^2\beta^2 d^4/(1+\chi)\kappa\mu$), non–linearity of fluid magnetization parameter ($M_3 = (1+M_0/H_0)/(1+\chi)$, Taylor number ($Ta = 4\Omega^2 d^4/\nu^2$), non–dimensional magnetic parameter ($M_2 = \mu_0 T_0 K^2/\rho_0 C_0(1+\chi)$, Prandtl number ($\Pr = \nu/\kappa$) and temperature dependent viscosity η. The typical value of M_2 for ferrofluids with different carrier liquids turns out to be of the order of $10^{-6} \ll 1$ and hence neglect the terms involving M_2 in (9).

The corresponding boundary conditions for the perturbed non–dimensional variables take the form

$$W = DW = \Theta = \Phi = \xi = 0 \quad \text{at } z = 0 \tag{12}$$

$$W = (1+\eta)D^2W + Ma\,a^2\Theta = D\Phi = D\Theta + Bi\,\Theta = D\xi = 0 \quad \text{at } z = 1 \tag{13}$$

where, $Ma = \sigma_T\Delta T d/\mu\kappa$ is the Marangoni number and $Bi = hd/k_t$ is the Biot number. The case $Bi = 0$ and $Bi \to \infty$ respectively correspond to constant heat flux and isothermal conditions at the upper boundary with respect to the perturbed temperature.

3 Method of Solution

Equations (8)–(11) together with boundary conditions (12) and (13) constitute an eigenvalue problem with thermal Rayleigh number R_t or Marangoni number Ma as an eigenvalue. To solve the resulting eigenvalue problem, weighted residual Galerkin method is used. Accordingly, the variables are written in a series of basis functions as

$$W=\sum_{i=1}^{n}A_i\, W_i(z),\ \Theta(z)=\sum_{i=1}^{n}C_i\,\Theta_i(z)$$

$$\Phi(z)=\sum_{i=1}^{n}D_i\,\Phi_i(z),\ \xi=\sum_{i=1}^{n}E_i\,\xi_i(z) \tag{14}$$

The eigenvalues have to be extracted from the above characteristic equation. For this, we select the trial functions as

$$W_i=z^{i+3}-\frac{5}{2}z^{i+2}+\frac{3}{2}z^{i+1},\ \Theta_i=z^{i+1}-2z^i, \tag{15}$$
$$\Phi_i=(z^2-11z/6+2/3)z^i,\ \xi_i=(z-1)^2z^{i+1},$$

these trial functions satisfy all the corresponding boundary conditions except the one, namely $D^2W+Ma\,a^2\Theta=0=D\Theta+Bi\,\Theta$ at $z=1$ but the residual from this equation is included as a residual from the differential equation. At this juncture, it would be instructive to look at the results for $i=j=1$ and for this order (14) gives the following characteristic equation

$$Ma=-\frac{61R_m a^2}{75600\delta_5}-2R_m<W\Theta> -\frac{(\delta_1+8\omega Pr)}{567000\,a^2<W\Theta>}\left[\frac{540Ta}{(\delta_2+2\omega)}+(63\delta_3+30\delta_4\,\omega)\right] \tag{16}$$

where,

$\delta_1=8a^2+20+15Bi,\ \delta_2=(2+\eta)(12+a^2),$

$\delta_3=-600(1+\eta)-180a^2\eta+a^4(90+50\eta),$

$\delta_4=216+19a^2$ and $\delta_5=(423+23M_3a^2)/3780.$

To examine the stability of the system, we take $\omega=i\omega$ in (16) and clear the complex quantities, we obtain,

$$Ma=-\frac{540Ta(\delta_1\delta_2+16\omega^2\,Pr)}{567000a^2<W\Theta>(\delta_2^2+4\omega^2)}+\frac{3(21\delta_1\delta_3-80\omega^2\,\delta_4Pr)}{567000a^2<W\Theta>} -2R_m<W\Theta>-\frac{61R_m a^2}{75600\delta_5}+i\,\omega\,\Delta \tag{17}$$

where,

$$\Delta=-\frac{1}{567000a^2<W\Theta>}\left[\frac{1080Ta(4\delta_2Pr-\delta_1)}{(\delta_2^2+4\omega^2)}+3(168\delta_3Pr+10\delta_1\delta_4)\right].$$

Since Ma is a physical quantity it must be real, so that it implies either $\omega=0$ or $\Delta=0$ (i.e. $\omega\neq0$) and accordingly the condition for steady and oscillatory onset is obtained.
The steady onset is governed by $\omega=0$ and it occurs at $Ma=Ma^S$, where

$$Ma^s=-\frac{\delta_1}{567000\,a^2<W\Theta>}\left[\frac{540Ta}{\delta_2}+63\delta_3\right] -2R_m<W\Theta>-\frac{61R_m a^2}{75600\delta_5}. \tag{18}$$

The oscillatory convection occurs at $Ma=Ma^0$, where

$$Ma^0=-\frac{(a_1a_4^2+a_2a_4+a_3)}{576000a_4\,a^2<W\Theta>}-2R_m<W\Theta>-\frac{61R_m a^2}{75600\,\delta_5} \tag{19}$$

Here, $a_1=9(\delta_1\delta_2-4Pr\,\delta_2^2)/32,$

$a_2=3(21\delta_1\delta_3+20Pr\,\delta_4\,\delta_2^2+720Ta\,Pr),$

$a_3=-43200Ta\,Pr\,\delta_4$ and $a_4=\dfrac{(5\delta_1\delta_4+84Pr\,\delta_3)}{5\delta_4(\delta_1-4Pr\,\delta_2)}.$

The corresponding frequency of oscillations is given by

$$\omega^2=-\frac{\delta_2^2}{4}+\frac{36Ta}{\delta_4}\left[\frac{1-2\beta_1 Pr}{1+2\beta_2 Pr}\right] \tag{20}$$

For the occurrence of oscillatory onset, ω^2 should be positive and the necessary conditions for the same are

$$Pr<\frac{(2a^2+4+3.75Bi)}{(2+\eta)(a^2+12)}\ \text{and}\ Ta>\frac{\delta_2^2\,\delta_4}{144}\left[\frac{1+2\beta_2\,Pr}{1-2\beta_1\,Pr}\right] \tag{21}$$

where,

$\beta_1=\dfrac{(2+\eta)(24+12a^2)}{20+8a^2+15Bi}$, and

$$\beta_2=\frac{42(a^4(90+50\eta)-600(1+\eta)-180\eta a^2)}{(20+8a^2+15Bi)(216+19a^2)}$$

From the relation (21), the value of $Pr<1$ for irrespective rate of a,η,Bi and thus it is evident that for the oscillatory onset to exist in the classical viscous liquids. However, the ferrofluids, whether it is water based or any other organic liquid based, Prandtl number is greater than unity and hence the overstability is not a preferred mode of instability. In what follows we restrict ourselves to the case of steady onset and put $\omega=0$ in (17). A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation

$$f(R_t,\ R_m,\ Ma,\ M_3,\ \eta, Bi,\ Ta,\ a)=0 \tag{22}$$

4 Results and Discussion

TABLE 1
COMPARISION OF CRIOTICAL VALUES OF R_{tc} AND R_{mc} FOR DIFFERENT VALUES OF Ma AND Bi WHEN $\eta=0$, $M_3=1$, AND $S=10^{-4}$

Bi	Ma	Present Analysis		Qin and Kaloni [8]	
		R_{tc}	R_{mc}	R_{tc}	R_{mc}
0	**0**	637.875	40.688	652.87	42.624
	30	416.358	17.335	414.72	17.199
	50	256.414	6.575	254.06	6.455
	60	172.539	2.977	171.44	2.939
	70	85.9213	0.738	85.67	0.734
	79.61	0.000	0.000	0.000	0.000
10	**0**	934.009	87.237	892.06	79.577
	100	748.641	56.046	721.01	51.981
	200	540.996	29.268	526.21	27.690
	300	306.831	9.414	301.89	9.114
	350	177.771	3.160	176.10	3.101
	413.44	0.000	0.000	0.000	0.000

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TABLE 2
COMPARISON OF Ma_c AND a_c FOR DIFFERENT VALUES OF Ta WHEN $R_m = 0$ AND $R_t = 0$

Ta	Vidal and Acrivos [30] Ma_c	a_c	Present study Ma_c	a_c
0	80	2.0	79.61	1.99
10^2	92	2.2	91.31	2.17
10^3	164	3.0	163.11	2.97
10^4	457	5.0	456.21	4.99
10^5	1400	8.6	1400.45	8.82

The linear stability theory is used to investigate the effects of rotation and temperature dependent viscosity on coupled Bénard–Marangoni ferroconvection in a horizontal ferrofluid layer. The lower boundary is taken to be rigid–isothermal, while the upper free boundary open to the atmosphere is flat and subject to a convective surface boundary condition. The resulting eigenvalue problem is solved by employing Galerkin weighted residual method with either thermal Rayleigh number (R_t) or Marangoni number (Ma) as the eigenvalue. Computations reveal that the convergence in finding critical Marangoni number Ma_c crucially depends on the value of Taylor number Ta. For higher value of Ta more number of terms in the expansion of dependent variables were found to be required. The results presented here are for i = j = 8 the order at which the convergence is achieved, in general.

In order to compare the results of the present analysis with those of Qin and Kaloni [8] obtained numerically, a new magnetic parameter $(S = \mu_0 K^2 \kappa \nu / (1+\chi)\rho_0 g^2 \alpha_t^2 d^4)$ which was introduced in the analysis. The critical values obtained for different values of Ma_c with values of magnetic parameter $S\,(=10^{-4})$ and Biot number $Bi\ (=0,\ 10)$ are exhibited in Table 1. In order to validate the numerical solution procedure used, first the critical values $(Ma_c,\ a_c)$ obtained from the present study under the limiting conditions are compared with the previously published results of Vidal and Acrivos [30] in Table 2. The results tabulated in Table 2 for different values of Ta are for $Bi = R_t = R_m = \eta = 0$ (i.e., classical Marangoni convec-

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TABLE 3
CRITICAL INSTABILITY PARAMETERS R_{tc} AND R_{mc} FOR DIFFERENT VALUES OF Ma AND Ta WHEN $\eta = 0.2$ AND $Bi = 2$

Ta	Ma_c	$R_m = 0$		$R_t = 0$							
				$M_3 = 1$		$M_3 = 15$		$M_3 = 25$		$M_3 \to \infty$	
		R_{tc}	a_c	R_{mc}	a_c	R_{mc}	a_c	R_{mc}	a_c	R_{mc}	a_c
0	**0**	**912.042**	**2.380**	**1159.552**	**2.441**	**964.544**	**2.456**	**946.102**	**2.434**	**912.042**	**2.380**
	50	**669.466**	**2.342**	**844.092**	**2.381**	**708.598**	**2.397**	**694.861**	**2.381**	**669.466**	**2.342**
	100	**407.267**	**2.330**	**504.740**	**2.351**	**431.087**	**2.363**	**422.713**	**2.353**	**407.267**	**2.330**
	150	**124.202**	**2.346**	**153.582**	**2.352**	**131.350**	**2.356**	**128.827**	**2.353**	**124.202**	**2.346**
	170.768	**0.00**	**2.361**	**0.00**	**2.361**	**0.00**	**2.361**	**0.00**	**2.361**	**0.00**	**2.361**
10^2	**0**	**1000.772**	**2.524**	**1266.806**	**2.594**	**1053.486**	**2.594**	**1034.73**	**2.573**	**1000.772**	**2.524**
	50	**761.108**	**2.484**	**955.918**	**2.530**	**801.707**	**2.537**	**8**	**2.521**	**761.108**	**2.484**
	100	**500.548**	**2.468**	**623.125**	**2.495**	**527.252**	**2.503**	**787.276**	**2.462**	**500.548**	**2.468**
	150	**217.706**	**2.479**	**268.399**	**2.489**	**229.152**	**2.493**	**517.748**	**2.489**	**217.706**	**2.479**
	185.837	**0.00**	**2.504**	**0.00**	**2.504**	**0.00**	**2.504**	**225.066**	**2.504**	**0.00**	**2.504**
10^3	**0**	**1595.941**	**3.233**	**1969.083**	**3.339**	**1652.752**	**3.284**	**1631.74**	**3.267**	**1595.941**	**3.233**
	50	**1374.820**	**3.193**	**1686.784**	**3.277**	**1423.718**	**3.238**	**2**	**3.223**	**1374.820**	**3.193**
	100	**1129.050**	**3.166**	**1376.392**	**3.228**	**1168.990**	**3.203**	**1405.63**	**3.191**	**1129.050**	**3.166**
	150	**856.251**	**3.156**	**1036.370**	**3.199**	**886.195**	**3.183**	**0**	**3.174**	**856.251**	**3.156**
	200	**553.491**	**3.166**	**664.700**	**3.190**	**572.488**	**3.183**	**1154.20**	**3.177**	**553.491**	**3.166**
	250	**216.951**	**3.199**	**258.385**	**3.207**	**224.201**	**3.205**	**7**	**3.203**	**216.951**	**3.199**
	279.582	**0.00**	**3.230**	**0.00**	**3.230**	**0.00**	**3.230**	**875.100**	**3.230**	**0.00**	**3.230**

tion for non–ferrofluids). From the values presented in Tables 1 and 2, it is evident that there is an excellent agreement between the results of the present study and the previously published ones. This verifies the applicability and accuracy of the method used in solving the convective instability problem considered.

The tight coupling between buoyancy, surface tension, magnetic and Coriolis forces is exhibited quantitatively by tabulating the values of triplets (R_{tc}, Ma_c, R_{mc}) for different values of Ta with $\eta = 0.2$ and $Bi = 2$ in Table 3. From the table, it can be seen that an increase in non–linearity of the fluid magnetization M_3 is to decrease R_{mc} but only marginally and thus it has a destabilizing effect on the stability of the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of M_3 increases. From the Table 3, we note that an increase in M_3 is to increase a_c and hence its effect is to decrease the dimension of convection cells. Besides, as M_3 increases, R_{mc} decreases and the results reduce to that of classical Bénard–Marangoni problem for ordinary viscous fluids as $M_3 \to \infty$. That is, $R_{mc} = R_{tc}$ as $M_3 \to \infty$.

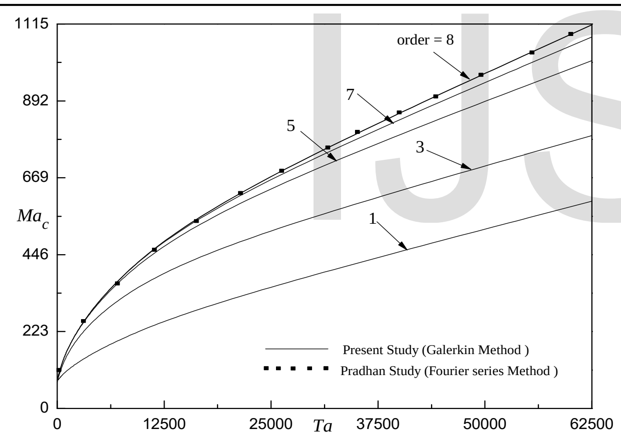

Fig. 1.Variation of Ma_c as a function of Ta for different orders by Galerkin method (present study) and for Fourier series method (previous study) when $M_3 = 1$, $R_m = 0$ and $Bi = 0$

It is instructive to know the process of convergence of result as the number of terms in the Galerkin approximation increases for the problem considered. Hence, the various levels of approximation to Ma_c and the corresponding a_c are also obtain for variation of Ta when classical Marangoni convection and results are shown graphically in Fig.1. It is seen that with an increase in the number of terms in Galerkin approximations, Ma_c goes on increasing and finally for the order $i = j = 8$ the present results converge compare well with results of previous study by Pradhan [31] and these results are obtained by Fourier series method. This clearly shows the accuracy of the numerical procedure employed in solving the problem.

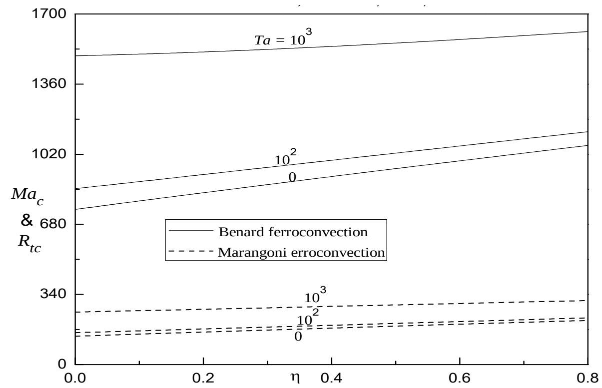

Fig. 2. Variations of Ma_c and R_{tc} as a function of η for different values of Ta when $M_3 = 1$, $R_m = 100$ and $Bi = 0$

In Figs. 2-4 are analogous to two types of ferroconvection, three solid curves corresponding to ferroconvection in the absence of surface tension force (i.e., Benard-ferroconvection, $Ma = 0$) and three dotted curves corresponding to ferroconvection in the absence of buoyancy force (i.e., Marnagoni-ferroconvection, $R_t = 0$). The plot of R_{tc} and Ma_c against η for various values of Ta for $M_3 = 1$, $R_m = 100$ and $Bi = 2$ with two types of ferroconvection. It shows that they are bridging the space between the Benard-ferroconvection and Marangoni-ferroconvection by increasing in Ta. Clearly, the results of Marangoni-ferroconvection advances the ferroconvection compared to Benard-ferroconvection. Fig. 2 reveals that the linear stability analysis can be expressed in terms of R_{tc} and Ma_c, the system with R_{tc} eigenvalue is unstable compared to Ma_c eigenvalue, it is noted that $Ma_c < R_{tc}$. Besides, it can be observed that an increasing η, the critical stability parameters (R_{tc} and Ma_c) increases and thus it has a stabilizing effect on the system. That is, the effect of increasing η is to delay the onset of ferroconvectionfor both the cases. This is the good agreement of the result found by Stengel et al. [32] and White and Perroux [33].

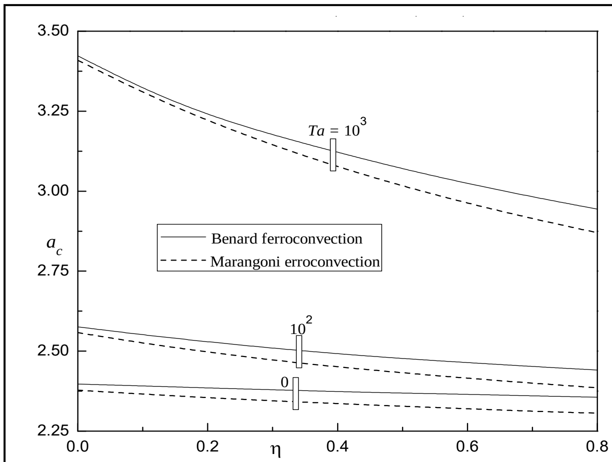

Fig. 3. Variations of a_c as a function of η for different values of Ta when $M_3 = 1$, $R_m = 100$ and $Bi = 2$

From Fig. 3, we note that increase in η is to decrease the critical wave number a_c and thus to widen the size of convection cells and opposite is the case with increasing Ta.

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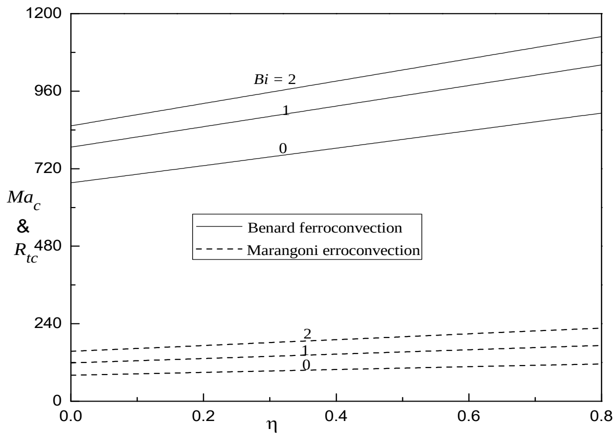

Fig. 4. Variations of (a) Ma_c and R_{tc} as a function of η for different values of Bi when $M_3=1$, $R_m=100$ and $Ta=100$

Fig. 4 shows that R_{tc} and Ma_c as a function of η for different values of heat transfer coefficient Bi (i.e., Biot number) with other parameters fixed ($M_3=1$, $R_m=100$ and $Ta=100$). From the figure it is evident that an increase in Bi is to increase R_{tc} as well as Ma_c. Thus, it is observed effect of Bi seen in Fig. 4 may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the upper surface and hence higher heating is required to make the system unstable.

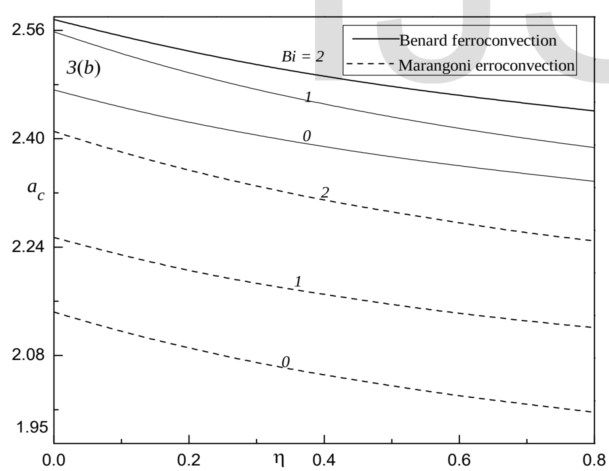

Fig. 5. Variations of a_c as a function of η for different values of Bi when $M_3=1$, $R_m=100$ and $Ta=100$

The variation of critical wave number a_c as a function of η is shown in Fig. 5 for different values of Bi with two types of ferroconvection. From the figure it is seen that the critical wave number a_c increases as the values of Biot number Bi increase and hence its effect is to contract the convection cell size.

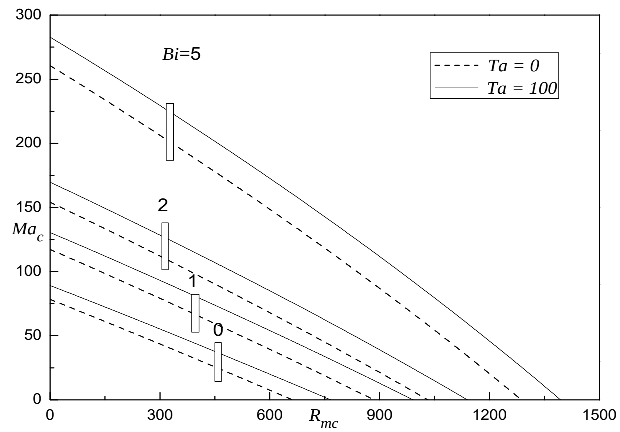

Fig. 6. Locus of Ma_c and R_{mc} for different values of Bi when $R_t=100$, $\eta=0.2$ and $M_3=1$

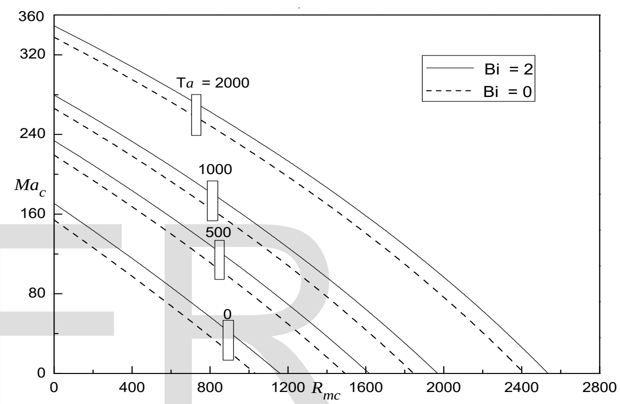

Fig. 7. Locus of Ma_c and R_{mc} for different values of Ta, $R_m=100$, $\eta=0.2$ and $M_3=1$

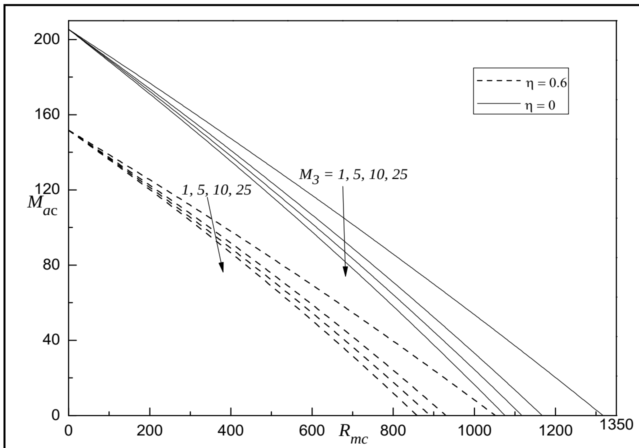

Fig. 8. Locus of Ma_c and R_{mc} for different values of M_3 when $Bi=2$, $Ta=100$ and $R_t=100$

Figs. 6, 7 & 8 show the locus of the critical Marangoni number Ma_c and the critical magnetic Rayleigh number R_{mc} for different Bi, Ta and M_3. Besides, from these figures, it is obvious that the curves are slightly convex and there is a strong coupling between Ma_c and R_{mc} and an increase in magnetic Rayleigh number has a destabilizing effect on the system.

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Thus, when the magnetic force is predominant, the surface tension force becomes negligible and vice–versa. From the figures, the extent to which the surface tension effect is diminished due to magnetic force however, depends on the Biot number, Bi, temperature dependent viscosity, η, non-linearity of the fluid magnetization, M_3, and strength of rotation, Ta. The critical magnetic Rayleigh number R_{tc} and Marangoni number Ma_c increase with an increase in Bi, Ta, η and decrease in M_3 thus the system is destabilizes.

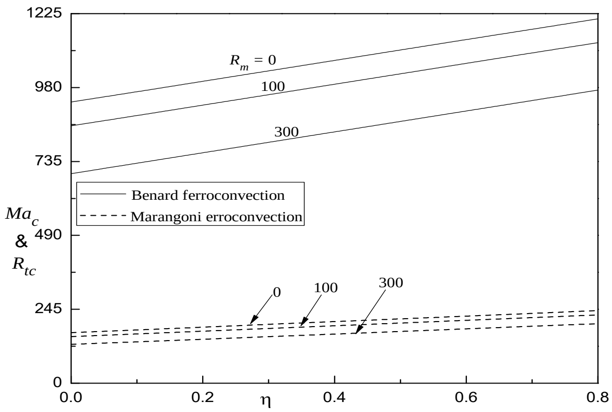

Fig. 9. Variations of Ma_c and R_{tc} as a function of η for different values of R_m when $M_3 = 1$, $Bi = 2$ and $Ta = 100$

The variations of R_{tc} and Ma_c as a function of η is shown in Fig. 9 for various values of R_m with two types of ferroconvection for $M_3 = 1$, $Bi = 2$ and $Ta = 100$. The amount of R_m is related to the importance of magnetic forces as compared to buoyancy forces. The case $R_m = 0$ corresponds to convective instability in an ordinary viscous fluid layer. From the figure, it is seen that an increase in R_m leads to decrease the values of R_{tc} and Ma_c suggesting that the ferrofluids carry heat more efficiently than the ordinary viscous fluids. This is due to an increase in the destabilizing magnetic force with increasing R_m, which favors the fluid to flow more easily.

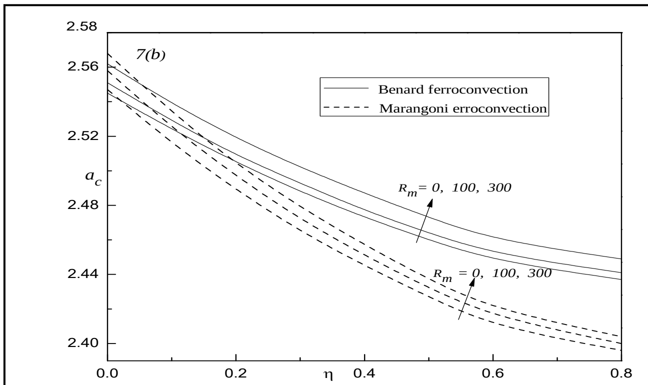

Fig. 10. Variations of a_c as a function of η for different values of R_m when $M_3 = 1$, $Bi = 2$ and $Ta = 100$

Fig. 10 illustrates that increase in the value of R_m is to decrease the critical wave number a_c slightly and thus to increase the size of convection cells.

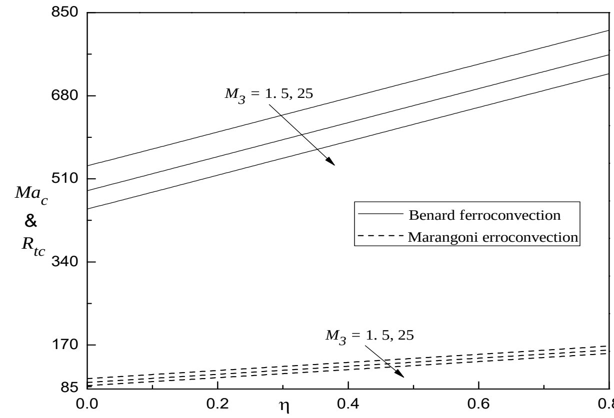

Fig. 11. Variations of Ma_c and R_{tc} as a function of η for different values of M_3 when $R_m = 100$, $Bi = 2$ and $Ta = 100$

Fig. 11 is the plot of Ma_c and R_{tc} versus η for different values of non-linearity of the fluid magnetization M_3. It is quite explicit that the effect of the departure from linearity in the magnetic equation of state, reflected by increasing in M_3, is to destabilize the system.

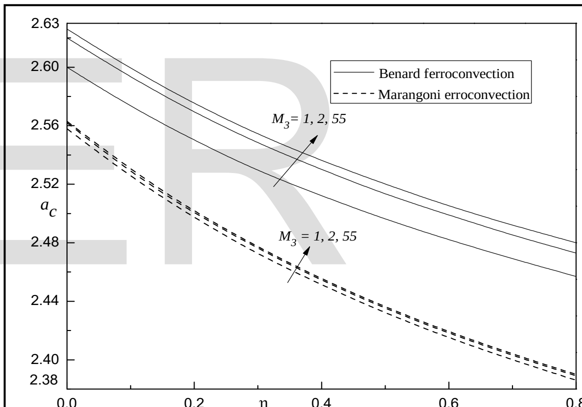

Fig. 12. Variations of a_c as a function of η for different values of M_3 when $R_m = 100$, $Bi = 2$ and $Ta = 100$

Fig. 12 represents the corresponding critical wave number a_c and it indicates that increase in M_3 is to increase a_c and thus their effect is to reduce the size of convection cells.

5 Conclusions

The combined effect of buoyancy and surface tension forces in a rotating ferrofluid layer heated from below subjected to temperature dependent viscosity is investigated theoretically. The lower boundary is taken to be rigid with fixed temperature, while the upper free boundary at which temperature–dependent surface tension effect is considered is non–deformable and subject to a general thermal condition. The Rayleigh–Ritz's method is employed to extract the critical stability parameters numerically with thermal Rayleigh number R_t or magnetic Rayleigh number R_m or Marangoni number Ma as the eigenvalue. The critical stability parameters R_{tc}, R_{mc} and Ma_c increases with an increase in Taylor number Ta, Biot number Bi and MFD viscosity parameter η thus

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their effect is to delay the onset of Bénard–Marangoni convection in a rotating ferrofluid layer. The effect of increasing the magnetic force R_m and the non–linearity of fluid magnetization M_3 is to suppress the onset of ferroconvection. The buoyancy force and surface tension force complement with each other and it is always found that $Ma_c < (R_{tc} \text{ or } R_{mc})$, a result in accordance with ordinary viscous fluids. As $M_3 \rightarrow \infty$, the results reduce to that of the Bénard–Marangoni problem for ordinary viscous fluids. The effect of increase in Ta and Bi as well as decrease in M_3 and R_m is to increase the critical wave number a_c and hence their effect is to decrease the dimension of convection cells.

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